

In triangle DAQ₁, $r = FA \cdot \tan(\text{FAQ}_1)$ and $r = FD \cdot \tan(\text{FDQ}_1)$

$$FA = \frac{r}{\tan(\text{FAQ}_1)} \text{ and } FD = \frac{r}{\tan(\text{FDQ}_1)}, \text{ but } FA + FD = AD$$

$$r \cdot \tan(\text{FDQ}_1) + r \cdot \tan(\text{FAQ}_1) = AD \cdot \tan(\text{FAQ}_1) \cdot \tan(\text{FDQ}_1), \text{ so } r = \frac{AD \cdot \tan(\text{FAQ}_1) \cdot \tan(\text{FDQ}_1)}{\tan(\text{FAQ}_1) + \tan(\text{FDQ}_1)}$$

Let AD=1 and let $\tan(m) = t$

$$\text{Angle DAE} = 90^\circ - 2m; \tan \text{DAE} = \tan(45^\circ - m); \tan(45^\circ - m) = \frac{1 - \tan(m)}{1 + \tan 45^\circ \cdot \tan(m)} = \frac{1 - t}{1 + t}$$

$$r = \frac{\frac{1 \cdot (1 - t)}{1 + t} \cdot t}{\frac{(1 - t)}{1 + t} \cdot t} = \frac{\frac{t \cdot (1 - t)}{(1 + t)}}{\frac{t \cdot (1 - t)}{(1 + t)}} = \frac{t \cdot (1 - t)}{1 + t^2} \dots\dots\dots [1]$$

$$\text{In triangle DCB, } CD = DE + EC = 2 \cdot \cos(2m) = 2 \cdot \frac{1 - \tan^2 m}{1 + \tan^2 m} = 2 \cdot \frac{1 - t^2}{1 + t^2}$$

$r = CG \cdot \tan(\text{GCQ}_2)$ and $r = DG \cdot \tan(\text{GDQ}_2)$

$$CG = \frac{r}{\tan(\text{GCQ}_2)} \text{ and } GD = \frac{r}{\tan(\text{GDQ}_2)}, \text{ but } CG + GD = CD$$

$$r \cdot \tan(\text{GDQ}_2) + r \cdot \tan(\text{GCQ}_2) = CD \cdot \tan(\text{GDQ}_2) \cdot \tan(\text{GCQ}_2), \text{ so } r = \frac{AD \cdot \tan(\text{GDQ}_2) \cdot \tan(\text{GCQ}_2)}{\tan(\text{GDQ}_2) + \tan(\text{GCQ}_2)}$$

$\angle \text{ACD} = 2m$; $\angle \text{CAJ} = \angle \text{CJA} = 90^\circ - m = \angle \text{Q}_2\text{JD}$; so $\angle \text{JQ}_2\text{G} = m$ and $\angle \text{Q}_2\text{AH} = 90^\circ - 3m$
 $\text{CQ}_2 = \text{AQ}_2$ and $\text{Q}_2\text{G} = \text{Q}_2\text{H} = r$ so $\angle \text{Q}_2\text{CG} = 90^\circ - 3m$

$$\tan(\text{DCQ}_2) = \tan(90^\circ - 3m) = \cot(3m) = \frac{1 - 3 \cdot \tan^2 m}{3 \cdot \tan m - \tan^3 m} = \frac{1 - 3t^2}{3t - t^3}$$

$$r = \frac{2(1 - t^2)}{(1 + t^2)} \cdot \frac{\frac{1}{t} \cdot \frac{(1 - 3t^2)}{(3t - t^3)}}{\frac{1}{t} \cdot \frac{(1 - 3t^2)}{(3t - t^3)}} = \frac{2(1 - t^2)(1 - 3t^2)}{(1 + t^2)(4t - 4t^3)} = \frac{(1 - 3t^2)}{2t(1 + t^2)} \dots\dots\dots [2]$$

Equating [1] and [2]: $\frac{t \cdot (1 - t)}{1 + t^2} = \frac{(1 - 3t^2)}{2t(1 + t^2)}$; $2t^2(1 - t) = 1 - 3t^2$ and $2t^3 - 5t^2 + 1 = 0$

or $2(t - 1)(t^2 - 2t - 1) = 0$ from which $t = 1/2$ or $t = 1 \pm \sqrt{2}$ and $m = 26^\circ 33' 54.2''$ and 135°

Using $t = 1/2$, $r = 1/5$; $\text{AE} = \sin(2m) = 0.8$ and $\text{DE} = \cos(2m) = 0.6$

Angle ACB = angle CAB = $180^\circ - 4m = 73^\circ 44' 23.3''$, making angle CBA = $32^\circ 31' 13.6''$
 AD must equal AC for the symmetry to exist, so

$$\frac{1}{\sin 32^\circ 31' 13.6''} = \frac{AB}{\sin 73^\circ 44' 23.2''} \text{ and } AB = 1.7857124 \text{ so } DB = 0.7857124$$

